Hydro-thermal coupling in a rough fracture

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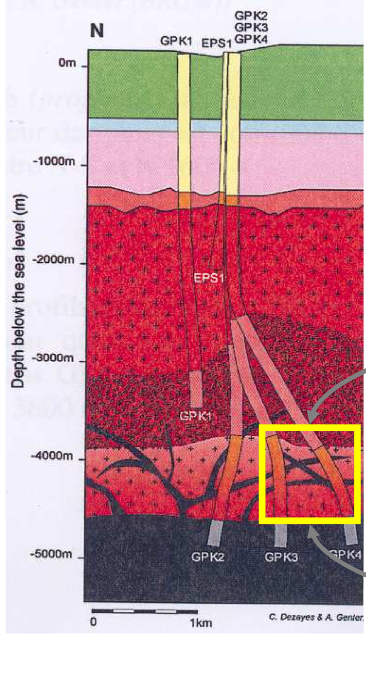
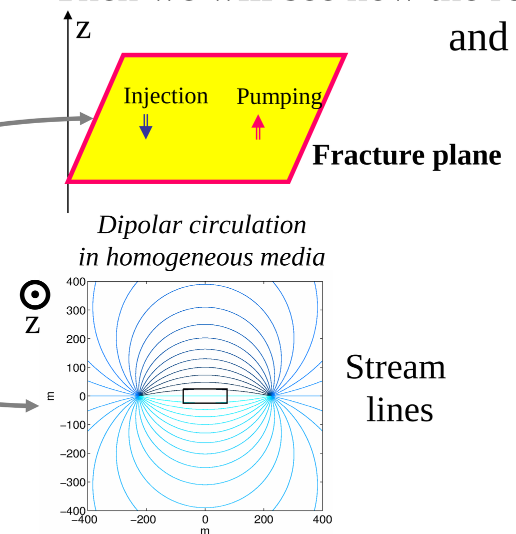
ABSTRACT

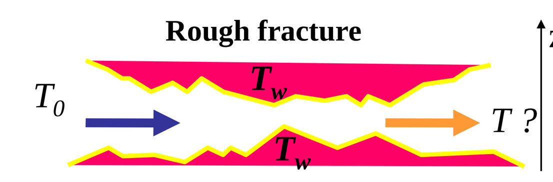
Heat exchange during laminar flow is studied at the fracture scale on the basis of the Stokes equation. We used a synthetic aperture model (a self-affine model) that has been shown to be a realistic geometrical description of the fracture morphology. We developed a numerical modeling using a finite difference scheme of the hydrodynamic flow and its coupling with an advection/conduction description of the fluid heat. As a first step, temperature within the surrounding rock is supposed to be hot and constant. Influence of the fracture roughness on the heat flux through the wall when a cold fluid is injected, is estimated and a thermalization length is shown to emerge. Our model shows that fracture roughness is responsible for channeling effects. Fluid flow is dominant in a significant subpart of the fracture where heat advection is important. Accordingly, temperature distribution is strongly affected by small fluctuations of the fracture aperture.

GEOTHERMAL EXPLOITATION



The context of study is an area where stream lines and isobars would be flat if the fracture were plane.
For example, it could take place in the framed area between injection and pumping wells.
Then we will see how the roughness of the fracture modify the hydraulic flux and the thermalization as well.





Impact of roughness on the fluid thermalization ?

METHOD

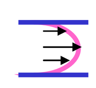
As reference case, the fracture is modeled by two parallel plates which are separated by a distance h with a pressure P_0 at the inlet and P_L at the outlet.

HYDRAULIC FLOW

- Permanent
- Laminarity

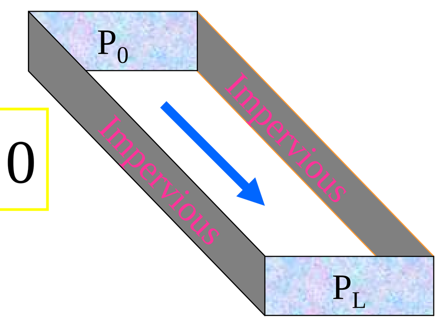
} Stokes : $\vec{\nabla} P = \eta \Delta \vec{v}$

Properties of fluid :
Density [kg/m³] : ρ
Conductivity [W/m/K] : λ
Specific heat capacity [J/kg/K] : c
Diffusivity [m²/s] : χ = λ / (ρc)

- Lubrication => locally smooth surface
  - Velocity $\vec{v} = \begin{pmatrix} v_x(x,y,z) \\ v_y(x,y,z) \\ 0 \end{pmatrix}$  Local parabolic law
  - Hydraulic flow $\vec{u} = \int_h \vec{v} dz = -\frac{h^3}{12\eta} \vec{\nabla} P$ Local cubic law
- Incompressibility : $\vec{\nabla} . \vec{u} = 0$

➢ ***Equation to be solved (2D) :***

$$\vec{\nabla}.(h(x,y)^3 \vec{\nabla} P) = 0$$



- Boundary conditions
  - Impermeability
  - $(P_L - P_0)$ imposed between the outlet and the inlet

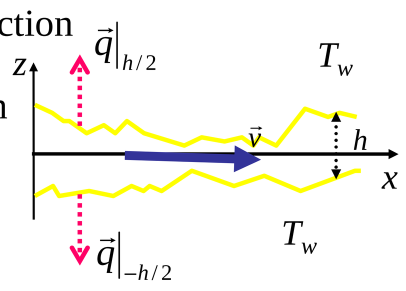
Averaging over thickness

ENERGY CONSERVATION

- Conduction $\vec{q} = -\lambda \vec{\nabla} T$
- Convection $\rho c \vec{v} T$

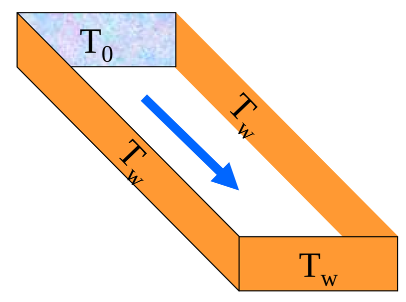
} $\Rightarrow \vec{v}.\vec{\nabla} T = \chi \Delta T$

- Assumptions :
  - * T_w constant and invariant
  - * $\vec{v} = \begin{pmatrix} v_x(x,y,z) \\ v_y(x,y,z) \\ 0 \end{pmatrix}$ => In plane convection
  - * Normal to plane conduction
  - * Lubrication



- Local temperature law
  $\Rightarrow T - T_w = \frac{f(x,y)}{h^2(x,y)} (a_4 z^4 + a_2 z^2 + a_0)$

2D-TEMPERATURE LAW



- Energy conservation $\bar{T}(x,y) = \frac{\int_h v(x,y,z) T(x,y,z) dz}{\int_h v(x,y,z) dz}$
- Boundary flow $j_w = -\lambda \frac{\partial T}{\partial z}\Big|_{h/2}$

➢ ***Equation to be solved (2D)***

$$h(x,y)\bar{\vec{v}}.\vec{\nabla}\bar{T} + 2\frac{\chi}{h(x,y)} Nu(\bar{T} - T_w) = 0$$

where $h(x,y)$, $\bar{\vec{v}}(x,y)$, $\bar{T}(x,y)$

Convection — Conduction

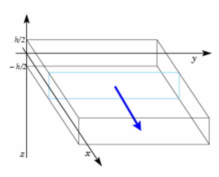
- Boundary conditions
  - $T = T_0$ at the inlet
  - $T = T_w$ far away from the inlet
  - $T = T_w$ on the walls

➢Nusselt number

$$Nu = \frac{-j_w}{q_{ref}} = \frac{70}{17} \approx 4.12 \quad \text{where} \quad q_{ref} = \lambda \frac{T_w - \bar{T}}{h}$$

REFERENCE CASE

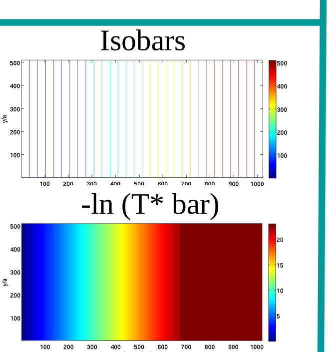
Analytic solutions



$$\bar{T}_{ref} - T_w = (T_0 - T_w) \exp\left(-\frac{x}{l_{ref}}\right)$$

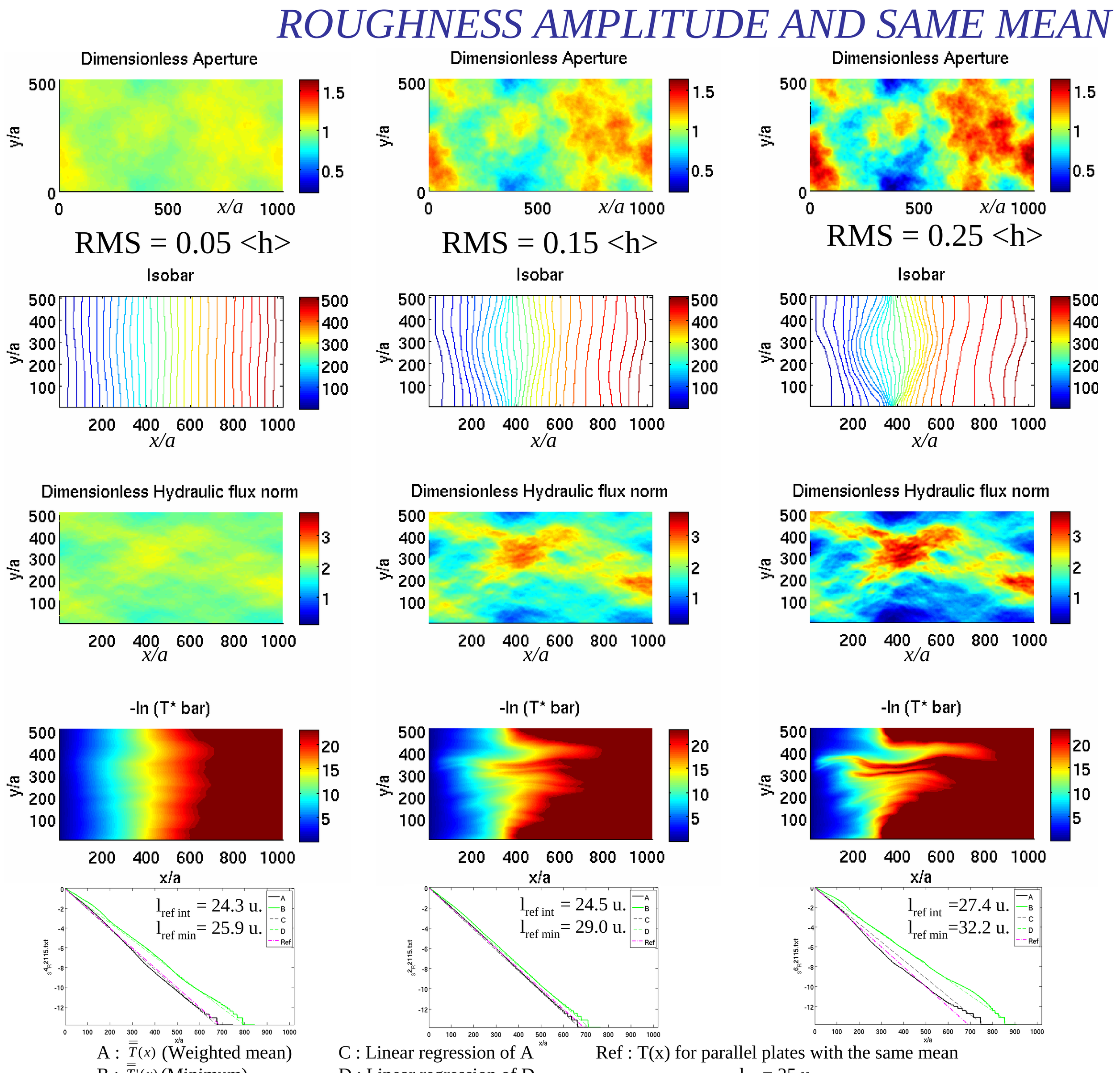
$$l_{ref} = \frac{h^2 \bar{v}}{2Nu\chi} = \frac{hPe}{Nu}$$

Peclet number $Pe = \frac{h\bar{v}}{2\chi}$



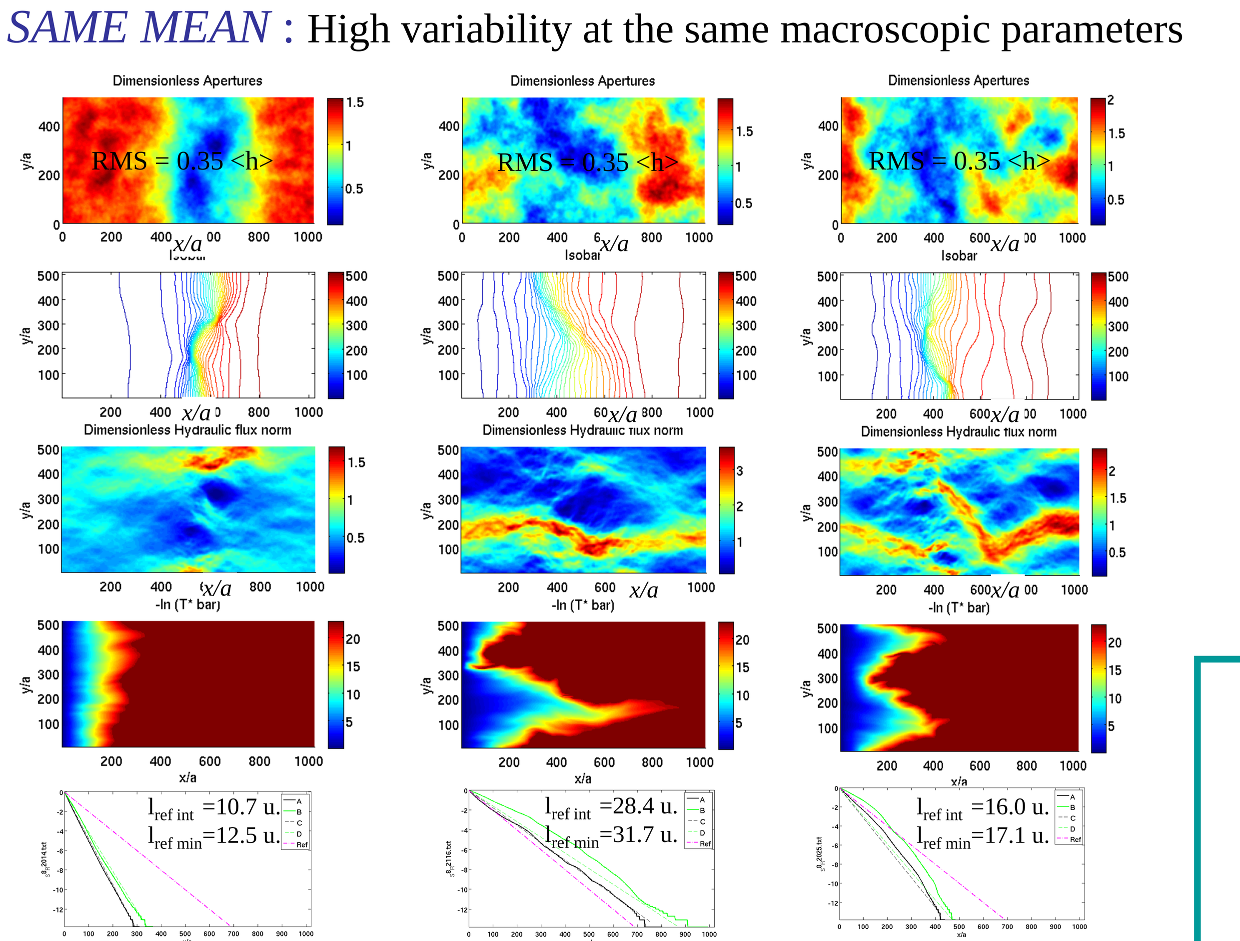
RESULTS

SAME REALIZATION WITH AN INCREASING ROUGHNESS AMPLITUDE AND SAME MEAN



A : $\bar{\bar{T}}(x)$ (Weighted mean) C : Linear regression of A Ref : T(x) for parallel plates with the same mean
B : $\bar{\bar{T}}'(x)$ (Minimum) D : Linear regression of D l_{ref} = 25 u.

VARIOUS REALIZATIONS FOR THE SAME RMS AND THE SAME MEAN : High variability at the same macroscopic parameters



A : $\bar{\bar{T}}(x)$ (Weighted mean) C : Linear regression of A Ref : T(x) for parallel plates with the same mean
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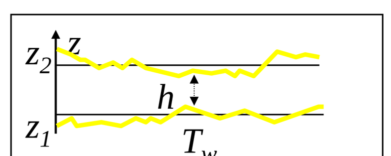
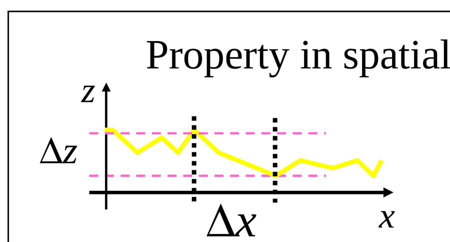
ROUGH FRACTURE : SELF AFFINE APERTURE



$h(x,y) = z_2(x,y) - z_1(x,y)$

$h^* = \frac{h}{h_{moy}}$

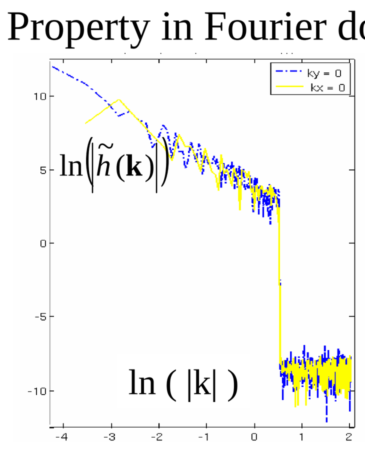
Property in spatial domain



$$\lambda^{\zeta} g\left(\lambda^{\zeta}\Delta h, [\lambda\Delta x, \lambda\Delta y]\right) = g\left(\Delta h, [\Delta x, \Delta y]\right)$$

Where *g* gives the probability to get a height difference *Δh* between two coordinate points separated by [*Δx*,*Δy*].

Property in Fourier domain



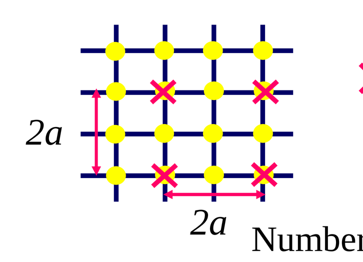
2D Fourier transform $\tilde{h}(\mathbf{k})$

$|\tilde{h}(\mathbf{k})| \approx |\mathbf{k}|^{-1-\zeta}$

ζ : Roughness exponent
ζ ≈ 0.8

METHOD OF RESOLUTION

- Discretization



● Apertures
✕ Temperatures, Hydraulic flux
a : Aperture mesh size

System length : $l_x > 20.l_{ref}$
Number of steps per reference length : $N_{mesh/lref} > 25$

$$\vec{\nabla}.(h(x,y)^3 \vec{\nabla} P) = 0$$

$$h^3_{i+0.5,j}(P_{i+1,j} - P_{i,j}) + h^3_{i-0.5,j}(P_{i-1,j} - P_{i,j}) + h^3_{i,j+0.5}(P_{i,j+1} - P_{i,j}) + h^3_{i,j-0.5}(P_{i,j-1} - P_{i,j}) = 0$$

$$h\bar{\vec{v}}.\vec{\nabla}\bar{T} + 2\frac{\chi}{h} Nu(\bar{T} - T_w) = 0$$

$$u^x_{i,j}(\bar{T}_{i+1,j} - \bar{T}_{i-1,j}) + u^y_{i,j}(\bar{T}_{i,j+1} - \bar{T}_{i,j-1}) + \frac{8a\,\chi\,Nu}{h_{i,j}}(\bar{T}_{i,j} - \bar{T}_w) = 0$$

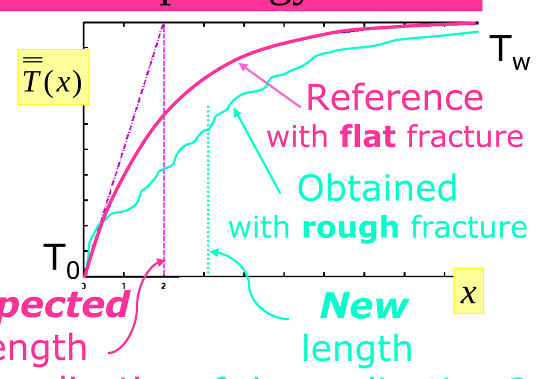
- Systems solved with the biconjugate gradient method
- Dimensionless variables
  - * Aperture : $h^* = \frac{h}{h_{moy}}$
  - * Pressures : $P^* = \frac{P - P_0}{2a.gradP_{macro}}$ where $gradP_{macro} = \frac{P_L - P_0}{2a.(n_x - 1)}$
  - * Hydraulic flux : $u^* = u\frac{48\eta}{gradP_{macro}\, h^3_{moy}}$
  - * Temperatures : $T^* = \frac{T - T_w}{T_0 - T_w}$

Range of validity

- ρ constant
  Low thermal dilatation coefficient
- z Conduction >> (x,y) Conduction
  Re >> 1.2
- x Conduction << x Convection
  Re >> 0.43
- Lubrication
  Re << 10²
- No heat source because of viscosity
  $Ec = \frac{\bar{v}^2}{c(T_w - T_0)} << 4.1\ 10^{-3}$
  Re << 4.10⁴ for ΔT=100 C

EQUIVALENT PROBLEM AT COARSENED SCALE

We aim at estimating the characteristic length of thermalization l_{ref}^{new} which depends on the fracture morphology



1D-temperature :

- From the energy conservation

$$\nabla\bar{\bar{T}} + \frac{(\bar{\bar{T}} - T_w)}{l_{ref}^{new}} = 0$$

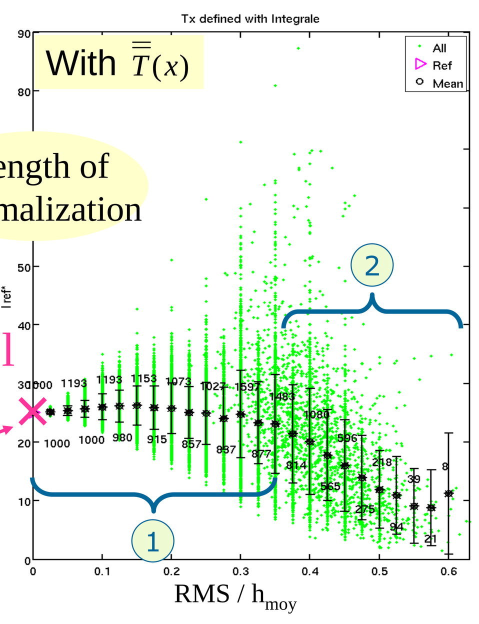
where $\bar{\bar{T}}(x) = \frac{\int_{l_y} \bar{v}_x(x,y)\bar{T}(x,y)dy}{\int_{l_y} \bar{v}_x(x,y)dy}$

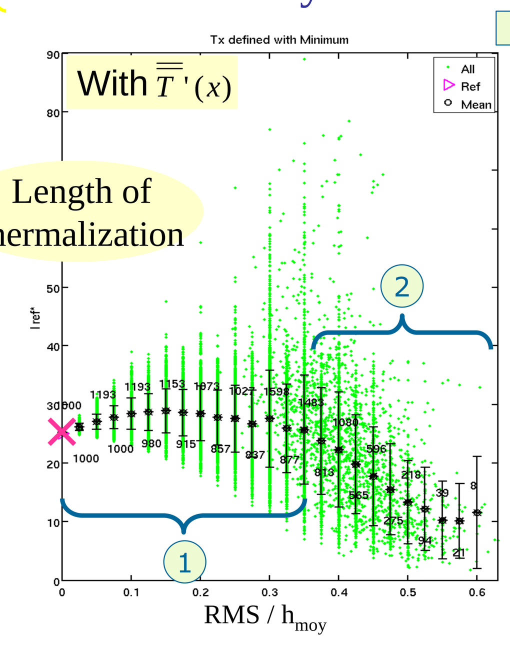
$\vec{u}(x,y) = h(x,y)\bar{\vec{v}}(x,y)$

- Or $\bar{\bar{T}}'(x) = \min_y \{\bar{T}(x,y)\}$

STATISTICAL RESULTS WITH

{ Various realizations
Various RMS
Same mean
Same size of system





CONCLUSION

- Channeling of
  - * Hydraulic flow
  - * Temperature
- Characteristic length of thermalization
  - * High variability !!
  - * Trend
    - ① RMS / $h_{moy} \in [0;0.35]$
      - ➢ Thermalization slightly slower
      - ➢ Thermalization disturbed by the roughness
    - ② RMS / $h_{moy} > 0.35$
      - ➢ Thermalization enhanced
      - ➢ Thermalization controled by small apertures

PROSPECTS

- Improving of the present study
  - * Hydraulic aperture influence ?
  - * Shape of the system ?
  - * Variation of Wall temperature
  - * Using ρ(T) dependance
- To provide a characteristic length for coarsened scale

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