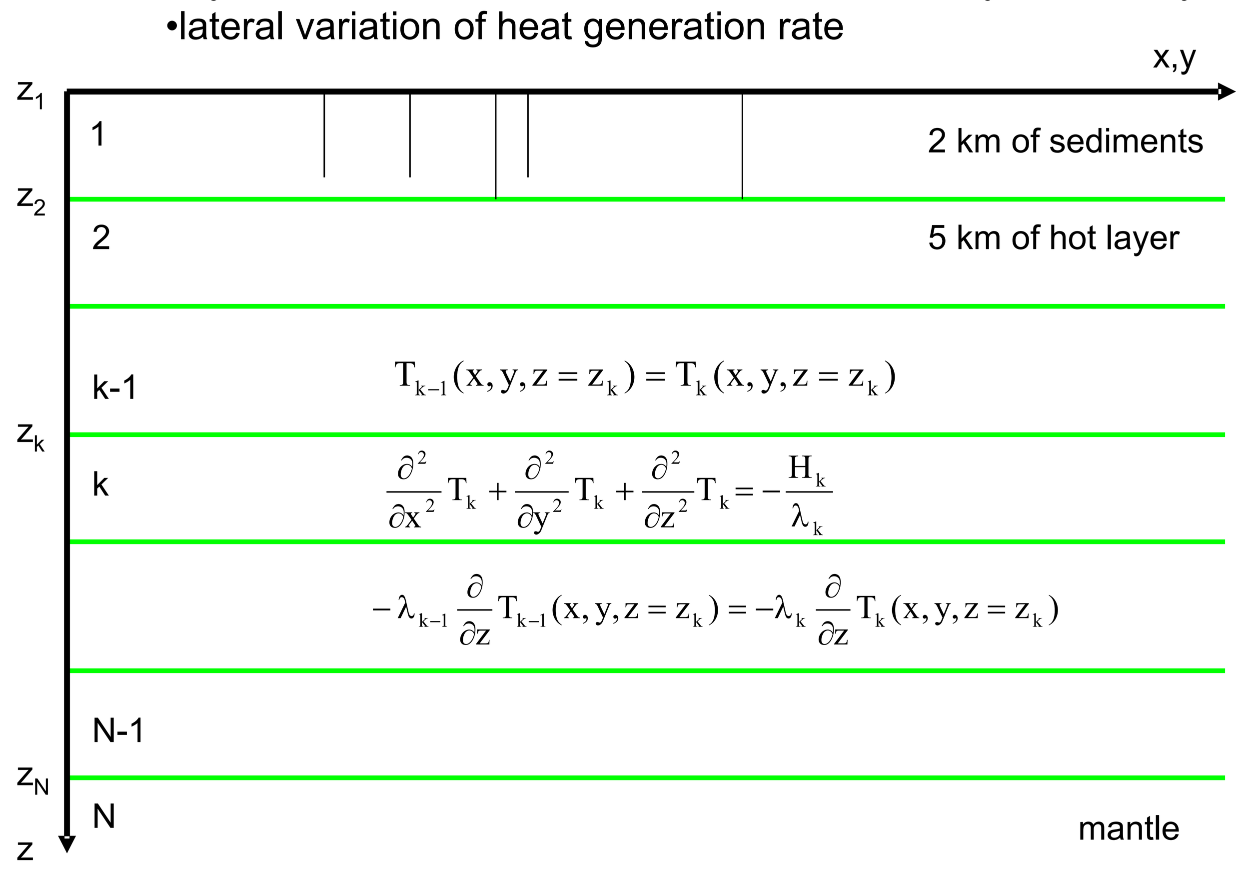
Subsurface thermal inference from simultaneous inversion of thermal information from many wells: Improving the geothermal field predictions

NIELSEN Søren, The University of Aarhus, Denmark
SLIAUPA Saulius, Institute of Geology and Geography & Vilnius University, Lithuania
STEPHENSON Randell, Faculty of Earth and Life Sciences, VU University Amsterdam, Netherlands
MOTUZA Gediminas, Vilnius University, Lithuania
CIURAITE Kristina, Vilnius University, Lithuania

THEORY

The model:

- layered Earth with constant thermal conductivity in each layer
- lateral variation of heat generation rate



$$T_{k-1}(x,y,z=z_k) = T_k(x,y,z=z_k)$$

$$\frac{\partial^2}{\partial x^2}T_k + \frac{\partial^2}{\partial y^2}T_k + \frac{\partial^2}{\partial z^2}T_k = -\frac{H_k}{\lambda_k}$$

$$-\lambda_{k-1}\frac{\partial}{\partial z}T_{k-1}(x,y,z=z_k) = -\lambda_k\frac{\partial}{\partial z}T_k(x,y,z=z_k)$$

Use a Fourier cosine representation of lateral variations in T and H:

Temperature
$$T_k(x,y,z) = \sum_{i=0}^{N_x-1}\sum_{j=0}^{N_y-1} T_{kij}(z)\cos\left(\frac{2\pi(x+\Delta x/2)}{2L_x}i\right)\cos\left(\frac{2\pi(y+\Delta y/2)}{2L_y}j\right)$$

Heat production
$$H_k(x,y) = \sum_{i=0}^{N_x-1}\sum_{j=0}^{N_y-1} H_{kij}\cos\left(\frac{2\pi(x+\Delta x/2)}{2L_x}i\right)\cos\left(\frac{2\pi(y+\Delta y/2)}{2L_y}j\right)$$

Equation for Fourier coefficients of temperature
$$\frac{\partial^2}{\partial z^2}T_{kij} = \left(\left(\frac{\pi}{L_x}i\right)^2 + \left(\frac{\pi}{L_y}j\right)^2\right)T_{kij} - \frac{H_{kij}}{\lambda_k}$$

Complete solution
$$T_{kij} = A_{kij}\exp(-\Omega_{ij}(z-z_k)) + B_{kij}\exp(+\Omega_{ij}(z-z_k)) - \frac{H_{kij}(z-z_k)^2}{2\lambda_k}$$
$$\Omega_{ij} = \sqrt{(\pi i/L_x)^2 + (\pi j/L_y)^2}$$

Fourier coefficients of layer k-1

$$\begin{pmatrix} A_{k-1ij} \\ B_{k-1ij} \end{pmatrix} = \begin{Bmatrix} \left(1+\frac{\lambda_k}{\lambda_{k-1}}\right)\exp(\Omega_{ij}d_{k-1}) & \left(1-\frac{\lambda_k}{\lambda_{k-1}}\right)\exp(\Omega_{ij}d_{k-1}) \\ \left(1-\frac{\lambda_k}{\lambda_{k-1}}\right)\exp(-\Omega_{ij}d_{k-1}) & \left(1+\frac{\lambda_k}{\lambda_{k-1}}\right)\exp(-\Omega_{ij}d_{k-1}) \end{Bmatrix} \begin{pmatrix} A_{kij} \\ B_{kij} \end{pmatrix} + \frac{H_{k-1ij}d_{k-1}}{2\lambda_{k-1}} \begin{pmatrix} \left(\frac{d_{k-1}}{2} - \frac{1}{\Omega_{ij}}\right)\exp(\Omega_{ij}d_{k-1}) \\ \left(\frac{d_{k-1}}{2} + \frac{1}{\Omega_{ij}}\right)\exp(-\Omega_{ij}d_{k-1}) \end{pmatrix}$$

Fourier coefficients of layer k

$$\Omega_{ij} = \sqrt{(\pi i/L_x)^2 + (\pi j/L_y)^2}$$

Inversion algorithm: Markov Chain Monte Carlo

$d_{cur} = m(p_{cur})$
$d_{cur:}$: data vector (borehole temperatures)
$p_{cur:}$: model parameter vector (heat productions, surface temperature)
m() : model-data relationship (layered Earth)

1) Calculate likelihood of current model (p_{cur}) by comparing model predictions to borehole data

2) Propose trial model: $p_{try:} = p_{cur:} + ?p$

3) Calculate likelihood of trial model (p_{try}) by comparing model predictions to borehole data

4) Draw random number 0 < u < 1

5) If u < likelihood of trial model / likelihood of current model

then replace current model with trial model

6) Goto 2)

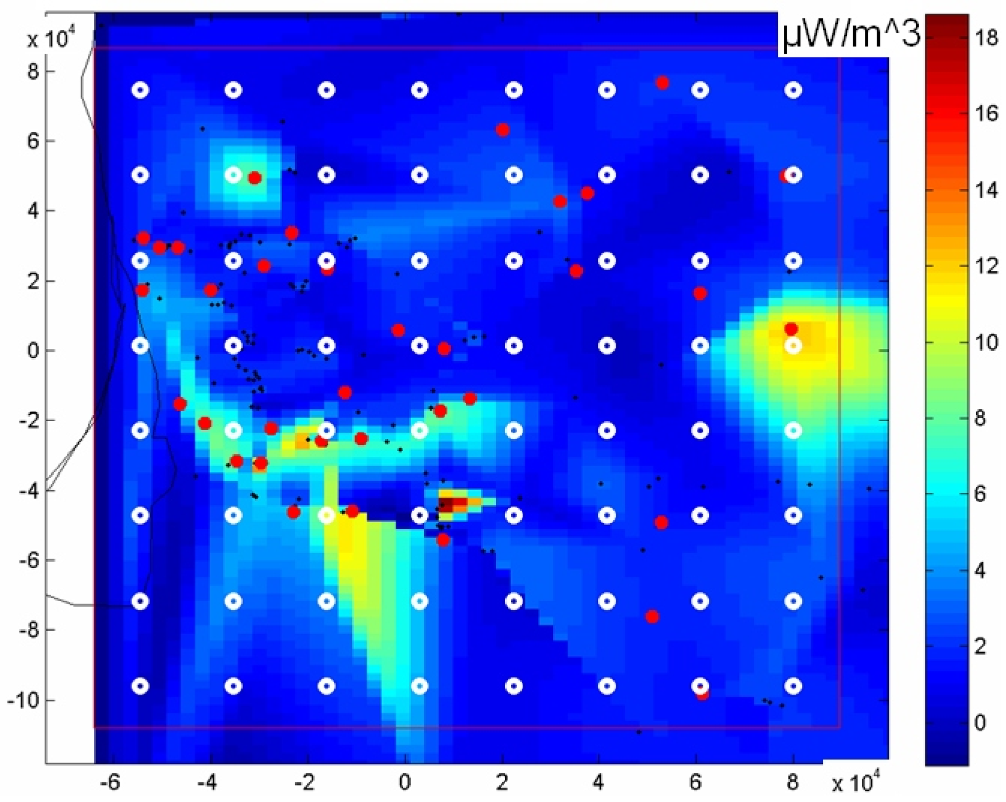
Conclusions

1) There is no need for a mantle plume to explain the thermal anomaly
2) Impulse response formulation of the 3D thermal model allows for very fast forward computation
3) This allows for use of the Markov Chain inversion method, which is very general and in principle allows for quantification of the joint probability density distribution of the solution to the inverse problem
4) Palaeo climate effects readily can be included
5) Borehole temperatures and basement heat generation data should be truely integrated in the inverse algoritm

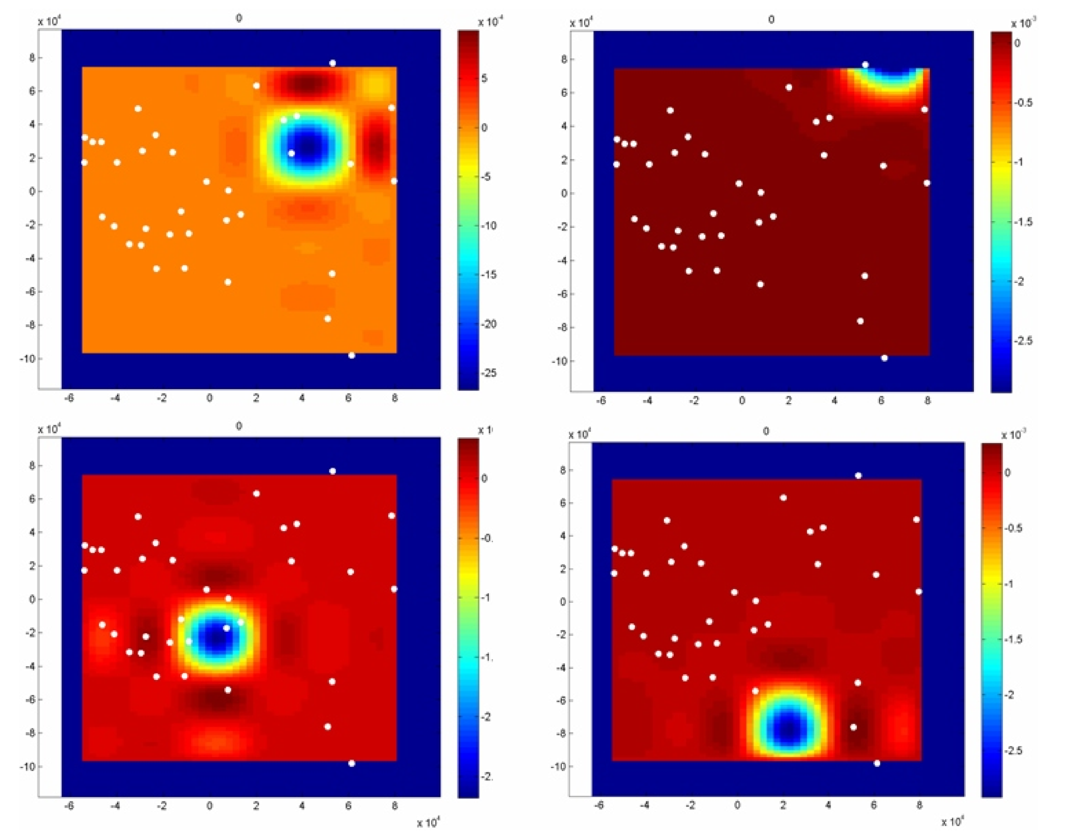
The use of exploration wells in geothermal studies is often made difficult by the poor quality of borehole temperature observations, which cannot be corrected for the influence of the drilling process, and the absence of proper thermal conductivity measurements. However, in many areas the density of wells is high enough that useful information can still be extracted. Here we present a flexible inverse formalism that allows analysing thermal parameters such as surface heat flux and basement heat production rate from exploration wells and to assess the uncertainty of subsurface thermal inference for use, for example, in terrestrial heat flow studies and geothermal energy planning.

Our quantitative model is based on three-dimensional heat conduction in a horizontally stratified subsurface. The solution is obtained using a two-dimensional Fourier cosine representation. The borehole data are modelled using the Markov Chain Monte Carlo method. This method is very flexible and provides histograms of the parameters of interest, including, for example, the surface heat flow or the temperature at the depth of planned geothermal reservoir.

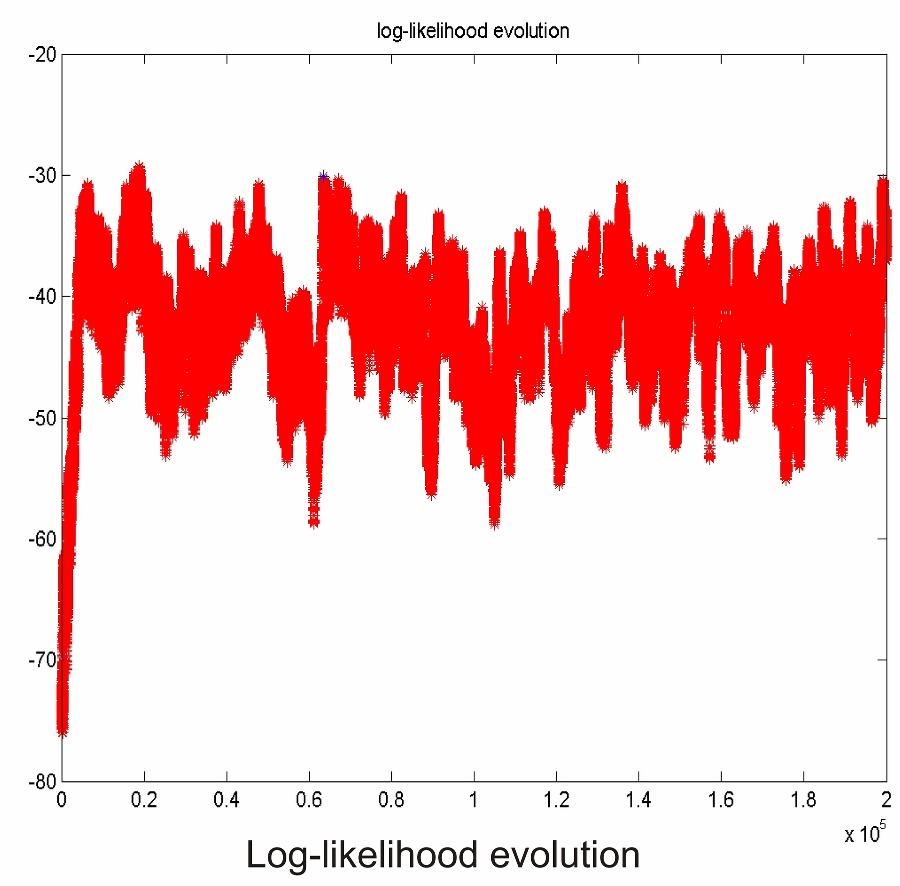
The West Lithuanian thermal anomaly displays a surface heat flow greater than 100 mW/m2 in a background East European Platform (EEP) heat flow of 50-60 mW/m2. This makes it one of the most profound heat flow anomalies of the EEP. It is located in an area of mainly early Palaeozoic platform sediments of thickness ~2 km overlying crystalline basement of Palaeoproterozoic age in western Lithuania and the Baltic Sea. The data available comprise borehole temperatures, basement heat production rates and estimates of thermal conductivity. The results show that the thermal anomaly can be explained by the thermal blanketing effect by sediments overlying a basement with an excessive heat production rate



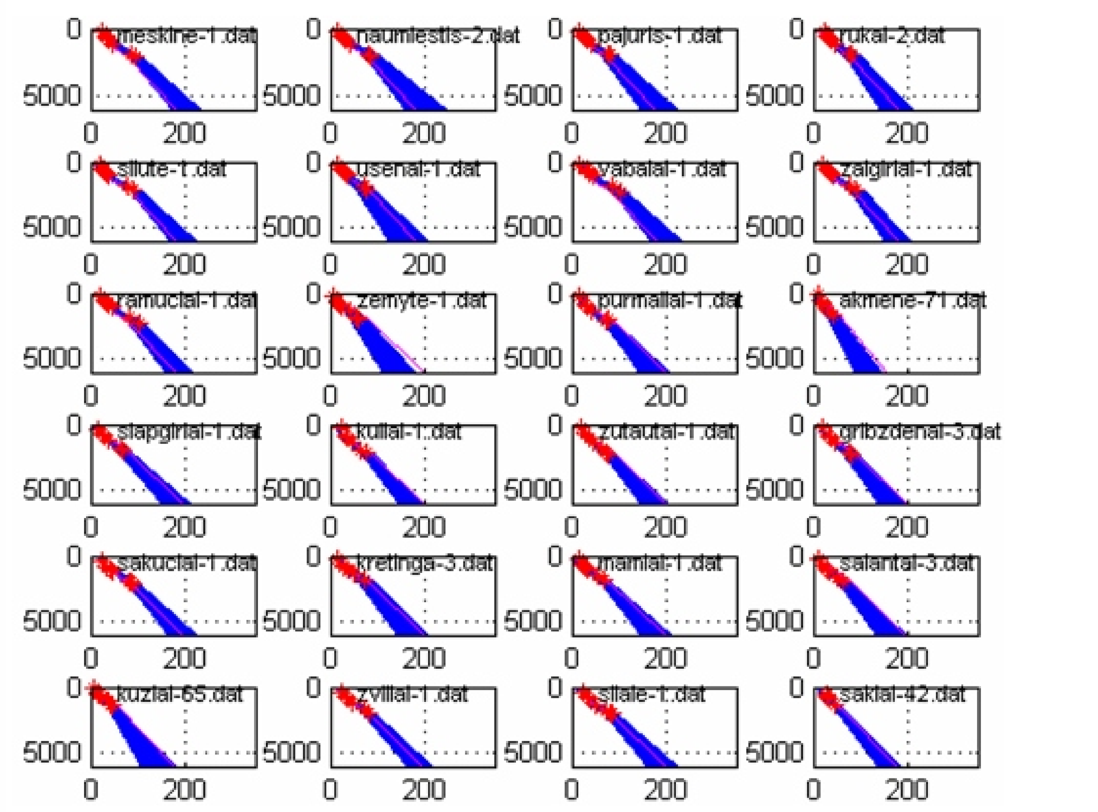
SURFACE HEAT PRODUCTION
Position of crustal heat production variables (white) and wells (red)



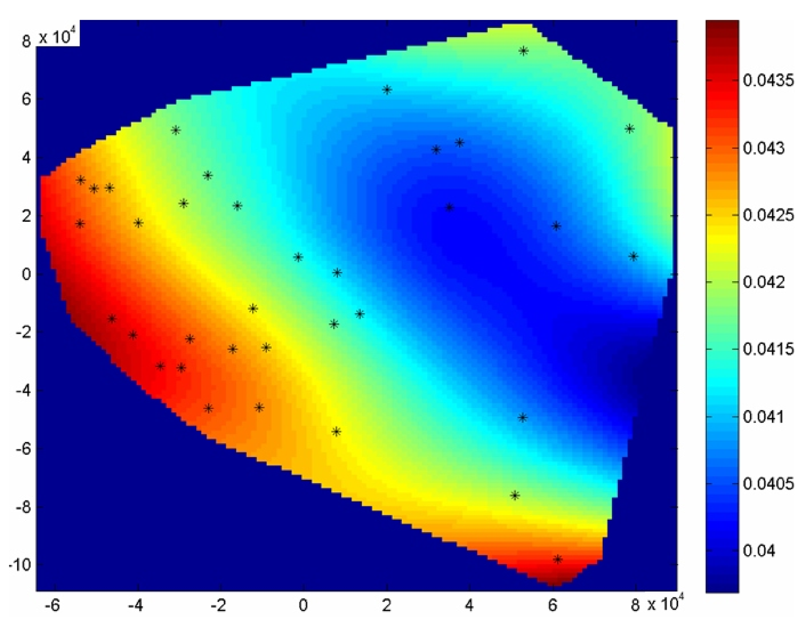
Use pre-calculated model impulse responses to obtain very fast forward response A total of 64 impulse response functions model the HF effect of the crustal layer



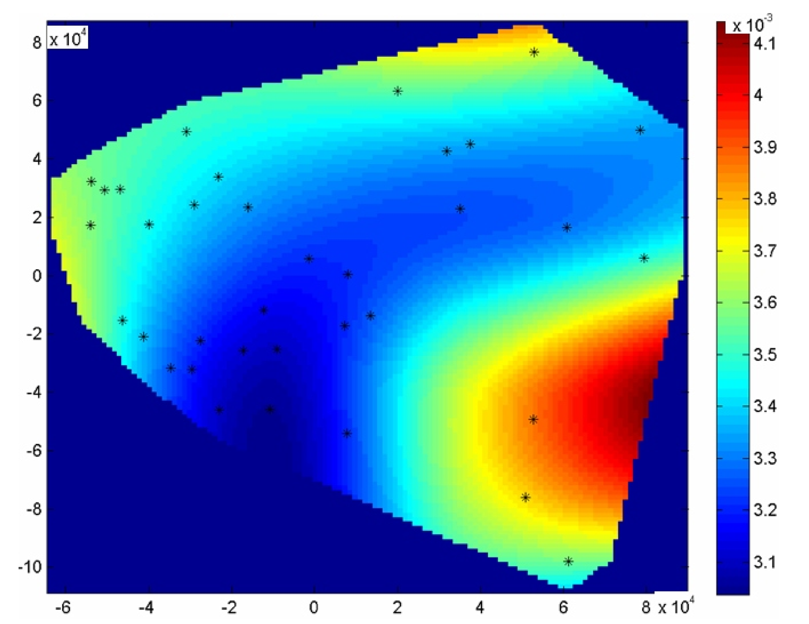
Log-likelihood evolution



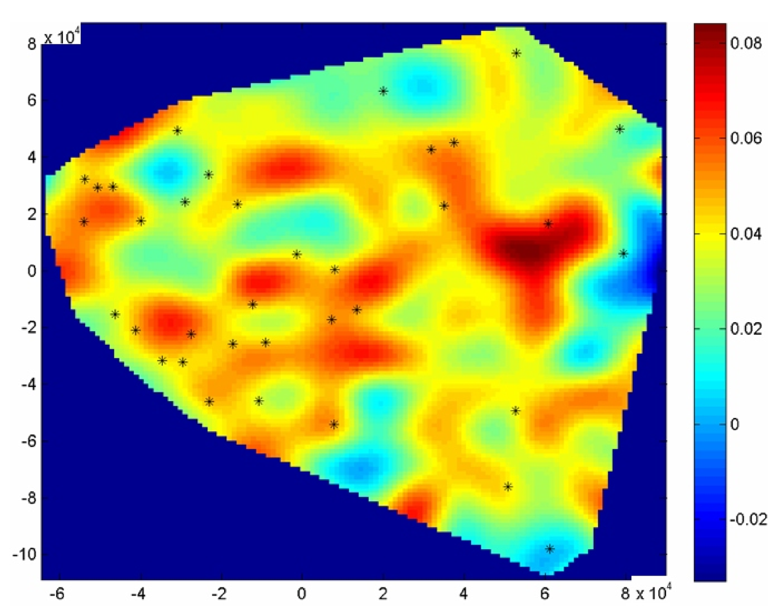
Some of the 200.000 models after the burn-in and their fit to data



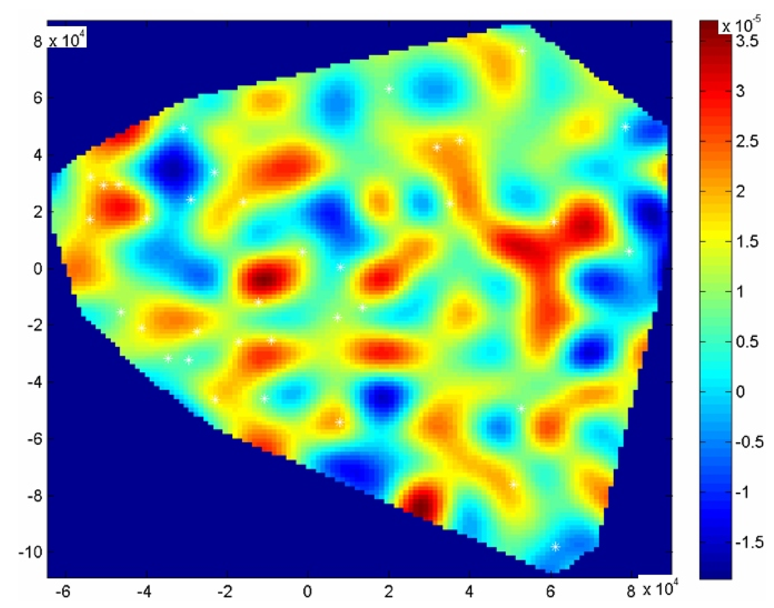
Best mantle heat flux



Standard deviation of mantle heat flux



Best crustal layer heat flux



Best crustal layer heat production rate